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GRADIENT CONTROL MAT SAFETY CONSIDERATIONS IN PIPELINE APPLICATIONS

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ABSTRACT

Gradient control mats are installed to protect workers from potentially hazardous voltages on a pipeline. Such voltages may be due to lightning and AC power system faults. The function of a gradient control mat is to limit the step potential between a worker's feet to a safe voltage level while standing on the mat and to limit the touch potential to a safe voltage level while standing on the mat and touching an above ground section of pipeline.

Potentials due to lightning are significantly more difficult to limit to safe levels than the potentials generated by power frequency faults, primarily due to the inductive voltage created by the very fast rate of rise (di/dt) of lightning current flowing through the inductance (L) of the mat and through connections to the mat. When safety concerns caused by lightning are addressed, the potentials associated with AC faults are also normally addressed, provided conductor the cross-section is adequate for the fault current that could be imposed. Only lightning effects are considered in the following analysis.

Some commonly used gradient control mat designs do not provide adequate safety from lightning, and some good gradient control mat designs are not being installed in a manner that provides the safety that they are otherwise capable of providing. This paper will illustrate the requirements for an effective gradient control mat design and important considerations in its installation.

KEY WORDS

Gradient control mats, ground mats, grounding mats, touch potential, step potential

INTRODUCTION

Two types of gradient control mats are commonly used; namely, a grid type mat or a single conductor spiral type mat. More recently, single conductor mats in a zigzag configuration have been introduced; however, these mats will have characteristics very similar to the single conductor spiral type mats to be analyzed. An analytical approach demonstrates why a grid type mat is vastly more effective than a single conductor type mat in limiting touch and step potentials due to lightning.

ASSUMPTIONS

To analyze and compare the two types of gradient control mats on a comparable basis, it is necessary to make certain assumptions. The assumptions used are listed below

1. For calculation purposes, a gradient control mat of each type will be constructed so that its outer edge extends out 5.5’ (1.68m) from the centerline of a 12” (305mm) diameter pipe projecting above ground. The inner diameter of each mat will be the same as the pipe diameter and this inner edge will be connected directly to the pipe.
2. Both mat types are assumed to be made of a material that can be directly connected to the pipe wall (e.g. zinc). The conductor in the single conductor spiral mat is assumed to have a resistivity at least equal to or less than that of #4 AWG copper which has a resistance of about 0.25 milliohms per foot. This value of resistance results in a resistive voltage drop that is negligible relative to the inductive voltage drop; thereby simplifying the calculations. The size of the zinc conductor used in the analysis was 0.50” x 0.562” (12.7mm x 14.3mm).
3. The radial distance between turns of the spiral mat is 12” (305mm), selected because this is a commonly recommended value[1].
4. The grid type mat is assumed to round, the same diameter as the spiral mat, and made of a 2” x 2”x 0.075” (51mm x 51mm x1 9mm) zinc mesh (12.5 gauge).
5. All current is assumed to flow through the single conductor of the spiral mat, and radially through the multiple current paths in the grid type mat before flowing to ground as this provides a direct comparison between the two mat types and represents the worst-case potentials. (In reality, current will be randomly dissipated into the earth below the mat as it flows through the conductor.)
6. Potential calculations will be based on two different values for the rate of rise (i.e., di/dt) of lightning currents, namely, di/dt = 15,000 A/μsec and 150,000 A/μsec. The larger value represents the typical rate of rise for a direct lightning strike and the lower value, (1/10 of the larger value), is considered more representative of typical lightning currents likely induced on a pipeline or a lower level direct strike. These di/dt values are slightly less than the 200,000 A/μ sec (1% limit) and 20,000 A/μ sec (50% value) for cloud-to-ground discharges[2]. The 1% limit means that the probability of the parameter being greater than the value shown is 0.01. The 50% value means that the parameter has an equal probability of being higher or lower than the value shown.

Figures 1 and 2 show the two mats analyzed. Each mat has an inner diameter of 12” (305mm), which is the same as the pipe outer diameter, and an outer diameter of 5.5’ (1.68m).

Although a grid type mat would normally be rectangular, the calculations will still apply out to the radial dimensions used in this analysis.

ACCEPTABLE TOUCH AND STEP POTENTIALS

While a North American standard exists for touch and step potentials at power frequency voltages, a similar standard is not known to exist for the voltages and currents produced by lightning. An International Electrotechnical Commission (IEC) Publication, 479-2, entitled “Effects of Current Passing Through the Human Body” deals with alternating current with frequencies above 100 Hz, for special waveforms of current, and for unidirectional single impulse currents of short duration. The maximum safe or tolerable voltage depends on multiple factors, including but not limited to:

- Exposure duration
- Current path through the body
- Body weight.
- Body contact resistance (e.g., highly influenced whether on native soil or high resistivity stone).

For a typical 5 cycle duration, 60 Hz AC fault, tolerable voltages[3] are considered to range from several hundreds of volts for a typical person standing on 50 ohm-meter native soil to several thousands volts if standing on high resistivity stone. Lightning voltage/current exposure durations will be orders of magnitude less than for any power frequency exposure; hence, the tolerable voltage magnitudes will be greater. However, from a practical standpoint, not many workers will likely be inclined to work on any gradient control mat that allows touch and step potentials much higher than those established for power frequency exposures. As the analysis will illustrate, the differences in protection offered to a worker between the two common gradient control mat designs analyzed is so great that attempting to define the maximum allowable touch and step potential due to lightning is not necessary.

TOUCH AND STEP POTENTIAL COMPARISONS

Calculations are shown in Appendix A for the grid type mat and in Appendix B for the spiral type mat. The results are tabulated in Tables 1, 2, 3, and 4. In Tables 1 and 2, columns 2 and 4 show the touch potentials for the two di/dt values used, and columns 3 and 5 show the corresponding step potential values.

As would be expected for a grid type mat, the touch potential increases with radial distance from the pipe, and the step potential decreases. As the current flows radially outward in the mat, there are increasingly more parallel current paths in which the current can flow, thereby decreasing the inductance of the current path, which reduces the voltage drop developed per unit of radial distance. The resistive voltage drop is negligible and for all practical purposes can be ignored based on the assumptions used.

By contrast, in a single conductor spiral mat the current has only one conductor path to flow through with a fixed inductance per unit length. Another factor unique to the spiral mat is that in addition to the inductance for any given turn there is also an added mutual inductance component to each turn from all other turns. The net result is that a spiral type mat has a significantly higher inductance current path, and therefore, a significantly higher touch and step potential than a grid type mat. The touch and step potential calculations for a spiral type mat are shown in Appendix B and the results shown in Table 2. Note, in Table 2 the voltage values are in kV and kV/m versus V or V/m in Table 1.

Both the touch and step potentials for the spiral mat are unacceptably high to be safe by several orders of magnitude. The voltage gradients calculated could not likely be achieved with a 12” (305mm) radial separation between turns because the soil would not likely support voltages of this magnitude. Even if one assumes that a substantial part of the available lightning current flows into the earth below the mat (thereby reducing the voltage gradient caused by current flowing in the spiral conductor), this current will cause a voltage gradient in the earth that will not be limited by the spiral mat due to the large voltage it allows turn-to-turn.

To further compare the two types of control mats, Table 3 tabulates the ratio of touch potentials and Table 4 the ratio of step potentials for the spiral versus grid mats analyzed using the data from Tables 1 and 2. For example, at 30” (762mm) from the pipe center, the touch potential for the spiral mat is 1,271 times (for di/dt = 15,000A/μsec) and 12,710 times (for di/dt = 150,000A/μsec) that of the grid type mat. Similarly, the step potential ratios of the spiral to grid type mat analyzed are illustrated in Table 4.

NACE guidelines[4] define step potential as the voltage difference between two points separated by one pace, assumed to be one meter, calculated in the direction of maximum potential gradient. Therefore, the step potentials tabulated in Tables 1 and 2 are those values given in V/m or kV/m.

The potentials associated with a grid type mat will always be several orders of magnitude lower than for any spiral, single conductor mat, thereby providing a significantly higher level of safety to a worker standing on the mat when providing safety from lightning caused voltage on a pipeline is of concern. The grid spacing also notably affects the touch and step potentials. For additional information, see the section in Appendix A entitled “Effect of Grid Size and Spacing.”

INSTALLATION CONSIDERATIONS

A very important consideration to obtain the effectiveness that a grid type gradient control mat can provide is to connect the mat to the pipe with the shortest, lowest inductance connection possible. A short, wide, thin bus is preferred to round conductor because the inductance will be on the order of 50% less. The length of this bus should be as short as possible, preferably several inches or less. To illustrate, if the gradient control mat were connected with 2’ (0.61m) of round lead, and one uses the same two values for di/dt used throughout this analysis and a typical lead inductance of 0.2×10^{-6} H/ft. (0.656×10^{-6} H/m), the potential of the mat with respect to the pipe is elevated by the voltage drop of the lead as follows. Instead of the mat touch potential starting at zero volts relative to the pipe, it starts at the potential created by the voltage drop in the connecting lead, which is calculated as follows:

$$V = L\ di/dt \qquad (1)$$

$$V = 0.2 \times 10^{-6} \times 2\text{ft.} \times 1.5 \times 10^{10}\ \text{A/sec} = 6{,}000\ \text{V (for di/dt} = 15{,}000\text{A/}\mu\text{sec)}$$

or

$$V = 0.2 \times 10^{-6} \times 2\text{ft.} \times 1.5 \times 10^{11}\ \text{A/sec} = 60{,}000\ \text{V (for di/dt} = 150{,}000\text{A/}\mu\text{sec)}$$

In the touch and step potential analysis, it was assumed that each gradient control mat was directly connected to the pipe wall at its inner diameter so this lead affect factor could be ignored. It has been observed that users of grid type gradient control mats will often connect a mat to the pipe as described in the above example, thereby largely or completely negating the benefits of the mat. The length of the connection between the pipe and the mat does not affect the step potentials because this is determined entirely by the parameters of the grid itself.

A second installation consideration is that a gradient control mat should be covered with about 4” to 6” of clean, washed, high resistivity crushed stone to provide an insulating barrier from the mat. This insulating barrier is desirable, but not essential, for a properly designed and installed grid type mat. However, this insulating barrier is very essential for a spiral type mat in that it will be the primary means of protection. The edges of the mat should be identified and workers should either be entirely over the mat or well away from the mat. The most dangerous place in the event of a lightning strike is to have one foot over the mat and one beyond the mat. With a crushed stone over-layer, the calculated potentials for the grid type mat, even with the higher of the two di/dt values used in the calculation, are considered safe in that they are well below the allowable potentials for power frequency faults [3], which are of considerably longer duration.

DECOUPLED GRADIENT CONTROL MATS

Whereas this analysis was based on using a mat material that could be bonded directly to a cathodically protected pipe, there are some advantages to decoupling the mat from the pipe as summarized below.

(1) The potential of the gradient control mat material is irrelevant when it is dc decoupled from the pipeline because a decoupled mat can not have any effect on the cathodic potentials.

(2) Any interaction between the gradient control mat material and the cathodic protection (CP) system is eliminated. (In certain types of soils, zinc can change characteristics; hence, changing how it interacts with the CP system.)

(3) A decoupled gradient control mat can be made from other less costly materials (e.g., galvanized steel).

(4) A decoupled gradient control mat enables instant-off pipeline potential readings to be taken in the vicinity of the mat, a consideration for some users. This cannot be done with directly connected zinc mats.

(5) Less CP current is required to obtain cathodic protection when no other metals are directly connected to the pipeline.

CONCLUSIONS

Where lightning caused voltages on pipelines are of concern, a properly designed and installed grid type gradient control mat can limit the touch and step potentials to values equal to or less than that allowed for power frequency voltages, thereby providing worker safety.

A spiral type gradient control mat, or other similar single conductor mat, cannot limit touch and step potentials due to lightning to safe levels due to its significantly higher inductance.

A grid type mat should be connected to the pipeline with the shortest, lowest inductance bus or lead feasible, preferably a few inches long, to minimize touch potentials. The longer the total length of the two connecting leads, the higher the touch potential. The length of the connection between the pipe and the mat does not affect step potentials for a grid type mat.

Any type mat should be covered with a 4” to 6” (102 to 152mm) layer of clean, high resistivity material. For any single conductor type mat, this material layer is the primary protection from lightning caused voltages, not the mat.

Where limiting potentials due to lightning is of concern, only a grid type mat should be considered.

Decoupling a gradient control mat offers certain advantages, namely, the option of using other mat materials, the ability to take instant off cathodic protection (CP) voltage readings in the vicinity of the mat, and the elimination of any interaction between the mat material and the CP system.

REFERENCES

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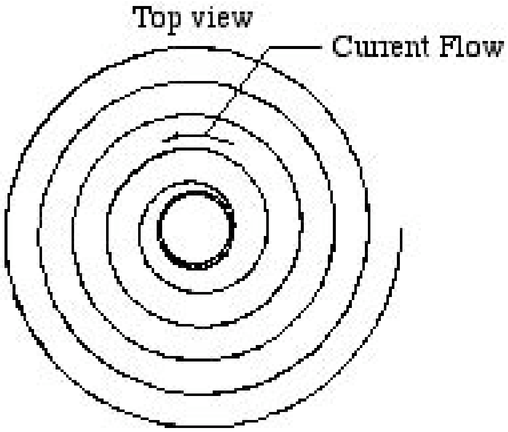

Figure 1
Spiral Type Gradient Control Mat

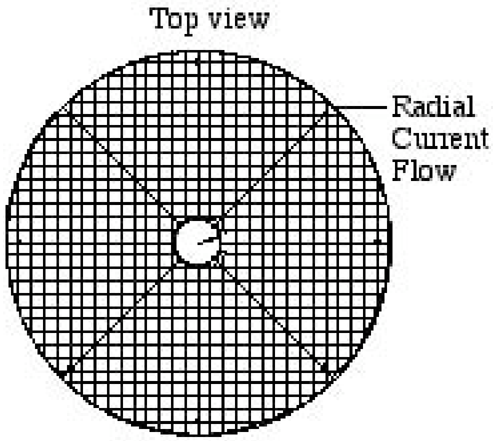

Figure 2
Grid Type Gradient Control Mat

Table 1
Touch and Step Potentials vs. Radial Distance From Pipe Centerline For the Grid Type Gradient Control Mat Analyzed

Radial Distance inches (mm)	Touch Potential, V di/dt=$1.5x10^{10}$ A/sec	Step Potential Volts/ft. (V/m)	Touch Potential, V di/dt =$1.5x10^{11}$ A/sec	Step Potential Volts/ft. (V/m)
6 (152.4mm)	0	0	0	0
18 (457.2mm)	57	57 (187)	570	570 (1,870)
30 (762mm)	83.4	26.4 (86.6)	834	264 (866)
42(1066.8mm)	101	17.6 (57.7)	1010	176 (577)
54 (1372mm)	114.5	13.5(44.3)	1145	135 (443)
66 (1676mm)	124.3	9.8 (35.2)	1243	98 (352)

Table 2
Touch and Step Potentials vs. Radial Distance From Pipe Centerline For the Spiral Type Gradient Control Mat Analyzed

Radial Distance inches (mm)	Touch Potential, kV di/dt =$1.5x10^{10}$ A/sec	Step Potential kV/ft. (kV/m)	Touch Potential, kV di/dt =$1.5x10^{11}$ A/sec	Step Potential kV/ft. (kV/m)
6 (152.4mm)	0	0	0	0
18 (457.2mm)	48.04	48.04 (157.6)	480.4	480.4 (1,576)
30 (762mm)	154.0	105.96 (347.5)	1540	1059.6 (3,475)
42(1066.8mm)	310.5	156.3 (512.7)	3105	1563.0 (5,127)
54 (1372mm)	506.7	196.3 (643.9)	5067	1961.3 (6,439)
66 (1676mm)	725.9	219.19 (718.9)	7259	2191.9 (7,189)

Table 3
Ratio of Touch Potentials for the Spiral versus Grid Type Gradient Control Mat

Radial Distance inches (mm)	Spiral/Grid Ratio for di/dt =$1.5x10^{10}$ A/sec	Spiral/GridRatio for di/dt = 1.5×10^{11} A/sec
6 (152.4mm)	0	0
18 (457.2mm)	843	843
30 (762mm)	1,847	1,847
42(1066.8mm)	3,074	3,074
54 (1372mm)	4,425	4,425
66 (1676mm)	5,840	5,840

Table 4
Ratio of Step Potentials for the Spiral versus Grid Type Gradient Control Mat

Radial Distance inches (mm)	Spiral/Grid Ratio for di/dt $=1.5\text{x}10^{10}$A/sec	Spiral/Grid Ratio for di/dt $= 1.5 \text{ x } 10^{11}$
6 (152.4mm)	0	0
18 (457.2mm)	843	843
30 (762mm)	4,014	4,014
42(1066.8mm)	8,881	8,881
54 (1372mm)	14,541	14,541
66 (1676mm)	22,366	22,366

APPENDIX A

Inductance/Voltage Calculations
For a
Grid Type Gradient Control Mat

Grid Resistance

The voltage developed as the result of current flowing through any conductor is dependent on the resistance and inductance of the material. When dealing with currents with a very fast rate of rise, such as lightning, the voltage developed by the resistive component will always be negligible relative to the inductive component for any grid type mat that would normally used; hence, the resistive component can be ignored.

Grid Inductance

To calculate the voltage gradient across a rectangular grid, the inductance of the grid (L_s) in henries per square must first be determined. The formula for this inductance is:

$$L_s = \mu_0 s/2\pi \ \ln(s/\pi d) \qquad (2)$$

where:
$\mu_0 = 4\pi \text{ x } 10^{-7}$
s = grid spacing in meters
d = grid wire diameter in meters
L_s = henries/square

After L_s is determined, the inductance across the grid (L_g) must be determined and this will depend on the shape of the grid. For the round grid shown in Figure 2, the inductance from the inner radius of the grid, where the current is injected, to any larger radius is calculated as follows:

$$L_g = L_s [\ 1/2\ \pi \times \ln (b/a)] \qquad (3)$$

where:
a = the inner radius of the grid
b = any larger radius out to the outer radius of the grid.

After L_g is determined, the voltage potential between any two values of “a” and “b” is determined by the formula:

$$V_{ab} = L_g\ di/dt \qquad (4)$$

The value of di/dt now must be in A/sec (instead of A/μsecond) to be compatible with formulas for a grid type mat. Using the same two values for di/dt previously used, but converted to A/sec, the values for di/dt become:

di/dt = 15,000A/μsec or 1.5×10^{10} A/sec

and

di/dt = 150,000A/μsec or 1.5×10^{11} A/sec

All of the necessary information and formulas now exist to make the calculations to determine the potentials associated with a grid type mat.

First, from formula (2) the value for L_s will be calculated for a 2” x 2”x 12.5 gauge (0.075”) grid as this will be required to determine the voltage gradients across the grid.

S = 2” x 1 meter/39.37” = 5.08×10^{-2} m
d = 0.075” x 1 meter/39.37” = 1.905×10^{-3} m
$L_s = (4\pi \times 10^{-7} \times 5.08 \times 10^{-2}/2 \times 3.14) \times \ln(5.08 \times 10^{-2}/3.14 \times 1.905 \times 10^{-3)}$
$L_s = 10.165 \times 10^{-9} \times \ln 8.49$
$L_s = 10.165 \times 10^{-9} \times 2.139 = 21.74 \times 10^{-9}$henries/square

Since formula (3) shows that the inductance versus radial distance from the pipe will be nonlinear, the potentials will also be nonlinear. To better display the results of these calculations, the inductance and the voltage will be calculated in radial increments of 12”(305mm). The calculated voltage at each radial distance will represent the touch potential with respect to the pipe and the difference in voltage between each 12” increment will represent the step voltage in volts/per foot of radial distance.

For the first increment where a = 6” (the outer diameter of the pipe/inner diameter of the mat) and b = 18”, L_g is calculated from formula (3) as follows.

$L_g = L_s$ x [1/2 π) x ln (b/a)]

$L_g = 21.74 \times 10^{-9}$ x[1/(2 x 3.14) x ln (18/6)

$L_g = 3.457 \times 10^{-9} \times 1.0986$

$L_g = 3.8 \times 10^{-9}$ henries

Next, from formula (4), the voltage between radial distance "a = 6" and "b = 18" is calculated for the two values of di/dt previously selected as follows.

$V_{ab} = L_g$ x di/dt

$V_{6\text{-}18} = 3.8\text{x } 10^{-9} \times 1.5 \times 10^{10} = 57$ volts @ b = 18" (457mm)

and

$V_{6\text{-}18} = 3.8 \times 10^{-9} \times 1.5 \times 10^{11} = 570$ volts @ b = 18" (457mm)

Calculations for all other increments (i.e., b = 30", 42", 54" and 66") are not shown, but were similarly calculated and tabulated in Table I. In Table I, the voltage difference between adjacent radial increments was also calculated and tabulated to obtain the step potential per meter.

EFFECT OF GRID SIZE AND SPACING

The touch and step potentials calculated vary directly with the grid inductance in henries per square (L_s). Therefore, one can calculate the grid inductance for various grid dimensions and then directly scale the results previously obtained. To illustrate, L_s will be calculated for several different grid dimensions to show how this will affect the potentials.

For Case 1, the grid will be 3/4" x 3/4" x 9 gauge (0.1144"), and for the second case a 6" x 6" by 10 gauge (0.1019"). The metric equivalent is 19mm x 19mm x 2.9mm for Case 1 and 152mm x 152mm x 2.6mm) for Case 2.

Case 1

$L_s = \mu_0 s/2\pi \ \ln(s/\pi d)$

where:
$s = 0.75" \times 1$ meter/39.37" $= 1.905 \times 10^{-2}$ m
$d = 0.1144" \times 1$ meter/39.37" $= 2.91 \times 10^{-3}$ m

$L_s = (4\pi \times 10^{-7} \times 1.905 \times 10^{-2}/2 \times 3.14) \times \ln(1.905 \times 10^{-2}/3.14 \times 2.91 \times 10^{-3})$

$L_s = 3.81\text{x } 10^{-9} \text{ x ln } (2.083)$

$L_s = 3.81 \text{ x } 10^{-9} \text{ x } 0.634 = 2.42 \text{ x } 10^{-9}$ henries/square

For a grid with these dimensions, the touch and step potentials calculated would be scaled by the ratio of L_s values for Case 1 versus the primary case. This ratio is $2.42 \text{ x } 10^{-9} /21.74 \text{ x } 10^{-9}$ or 0.11. Therefore, all potentials shown in Table I can be multiplied by 0.11 to obtain the potentials for a grid of the dimensions used in Case 1.

Case 2

$L_s = \mu_0 \, s/2\pi \;\; \ln(s/\pi d)$

where

$s = 6" \text{ x 1 meter}/39.37" = 15.2 \text{ x } 10^{-2}$ m

$d = 0.1019" \text{ x 1 meter}/39.37" = 2.59 \text{ x } 10^{-3}$ m

$L_s = (4\pi \text{ x } 10^{-7} \text{ x } 15.2 \text{ x } 10^{-2}/2 \text{ x } 3.14) \text{ x ln}(15.2 \text{ x } 10^{-2}/3.14 \text{ x } 2.59 \text{ x } 10^{-3)}$

$L_s = 30.4 \; 10^{-9} \text{ x ln } 18.69$

$L_s = 30.4 \text{ x } 10^{-9} \text{ x } 5.13 = 156 \text{ x } 10^{-9}$ henries/square

For a grid of these dimensions, the touch and step potentials calculated would be scaled by the ratio of L_s values or $156 \text{ x } 10^{-9} /21.74 \text{ x } 10^{-9} = 7.2$. Therefore, all potentials shown in Table I would be multiplied by 7.2 to obtain the potentials for Case 2.

APPENDIX B

Inductance/Voltage Calculations for a Spiral Type Gradient Control Mat

Grid Resistance

One of the assumptions was that the single conductor spiral mat was made of a conductor with a resistance equal to or lower than #4 AWG copper, which is about 0.25 mΩ/ft. With this value of resistance, even with a 100,000 A peak lightning current, the resistive voltage drop would only be 25 volts/ft (82v/m). As will be seen in the calculations to follow, this is completely negligible compared to the inductive voltage drop; hence, it will be ignored. In the following calculations, the conductor size used was 0.598" (15.2mm) in diameter which is equivalent in cross section to a commonly used zinc conductor size for spiral gradient control mats, namely, 0.562" x 0.50" (14.3mm x 12.7mm). Hence, "a" in the following formula is 0.299" (7.595mm).

Grid Inductance

For a single conductor spiral type gradient control mat, the self inductance (L_{SI}) inductance of a single loop of radius "b" with wire diameter "a" is given by the formula:

$$L_{SI} = b\mu_0 \left\{ \left[\ln\left(\frac{8b}{a}\right) - 2 \right] + \frac{1}{4} \right\} \tag{5}$$

The mutual inductance (L_{MI}) from one loop at radius "b" and another loop at radius "c" is given the formula:

$$L_{MI} = \mu_0 \left\{ \frac{b+c}{\sqrt{bc}} \right\} \left[\left\{ 1 - \frac{2bc}{(b+c)^2} \right\} K \left\{ \frac{4bc}{(b+c)^2} \right\} - E \left\{ \frac{4bc}{(b+c)^2} \right\} \right] \tag{6}$$

where K and E are complete elliptical integrals.

If we use the effective loop radii given below for self inductance calculations and add the mutual inductance for the other loops, the following table results.

Table 5

Inductance Per Loop and Total Inductance for Spiral Mat

Loop No.	R(meters)	L_{SI}(μH)	L_{TOTAL}(μH)	$L_{SPIRAL\ TOTAL}$ (μH)
1	0.30899	1.56716	3.20282	3.20282
2	0.611568	3.62637	7.06411	10.26693
3	0.915698	5.89423	10.4356	20.70253
4	1.22017	8.29422	13.0750	33.77753
5	1.52477	10.7918	14.6128	48.39033

Total Spiral Inductance = 48.3903 micro henries

To calculate the loop radiuses shown in the above table, the following formulas were used.

$$r_0 = 6 * 0.0254m$$
$$r_f = 66 * 0.0254m$$

$$f = \frac{r_0}{\pi} \tag{7}$$

$$s(\theta) = f\left\{ \left[\frac{\theta}{2}\sqrt{\theta^2 + 1} + \frac{1}{2}\ln\left(\theta + \sqrt{\theta^2 + 1}\right) \right] \right\} \tag{8}$$

$$r(\theta) = \frac{s(\theta) - s(\theta - 2\pi)}{2\pi} \tag{9}$$

$$\theta = 3\pi, 5\pi, 7\pi, 9\pi, 11\pi$$

In Table 5, L_{TOTAL} is the inductance of a given loop of the spiral which includes the mutual inductance from all other loops. If one multiplies this loop inductance by di/dt, then you get the step potential (i.e., the voltage from one end of a single loop of the spiral to the other end of that loop of the spiral). Since these loop ends are 12" (305mm) apart, this becomes the step potential in volts/foot. The step potentials calculated in this manner are tabulated in Table 2 for the two different values of di/dt used in the analysis. To illustrate, the voltage developed at the end of the first loop or spiral would be:

$$V_{LOOP\,\#} = L_{TOTAL}\, di/dt \qquad (10)$$

$$V_{LOOP\,1} = 3.20282 \text{ x } 10^{-6} \text{ x } 1.5 \text{ x } 10^{10} = 48{,}0420 \text{ volts} = 48.04\text{kV}$$

or

$$V_{LOOP\,1} = 3.20282 \text{ x } 10^{-6} \text{ x } 1.5 \text{ x } 10^{11} = 480{,}420 \text{ volts} = 480.4\text{kV}$$

The step potentials for each loop were similarly for each di/dt value and tabulated in Table 2, columns three and five.

The touch potential versus radial distance from the pipe is determined by multiplying the total spiral inductance for each loop, $L_{SPIRAL\ TOTAL}$, times the two different values used for di/dt. For loop 2, at radial distance 30" (762mm) the touch potential would be:

$$V_{LOOP\,2} = L_{SPIRAL\ TOTAL}\, di/dt$$

$$V_{LOOP\,2} = 10.26693 \text{ x } 10^{-6} \text{ x } 1.5 \text{ x } 10^{10} = 154\text{kV}$$

or

$$V_{LOOP\,2} = 10.26693 \text{ x } 10^{-6} \text{ x } 1.5 \text{ x } 10^{11} = 1540\text{kV}$$

Touch potentials for all loops were similarly calculated and tabulated in Table 2, columns 2 and 4.